Question 0 – Solution

(30 points)

Introduction

YTP Contest has started!
Wi-Fi setting, check!
YTP contest URL, check!
Login and Submit button, check!
Get input values from STDIN, and return output to STDOUT.

Everything ready! 30 points is coming!

Go get it!!

Description

Coding for A + B = C.

Input Format

A B

One line, two integers A B, separated by white space.

Output Format

C

C is an integer.

Data Range

A, B, C < 9999999.

Data Examples

Input Example 1

2756 707

Output Example 1

3463

Input Example 2

```
23024 19109
```

Output Example 2

```
42133
```

Input Example 3

```
6253 14484
```

Output Example 3

```
20737
```

Example Explanation:

The summation of 2756 and 707 is 3463.

Sample Codes:

Here following sample codes for c, C++, Java, and Python:

[Sample Code: C]

```
#include <stdio.h>

int main() {

     int a, b;
     scanf("%d %d", &a, &b);
     printf("%d\n", a+b);
     return 0;

}
```

[Sample Code: C++]

```
#include <iostream>

int main() {

     int a, b;
     std::cin >> a >> b;
     std::cout << a + b << std::endl;
```

```
    return 0;

}
```

[Sample Code: Java]

```
import java.util.Scanner;

public class solution {
    public static void main(String args[]) {
        Scanner in = new Scanner(System.in);
        int a, b;
        a = in.nextInt();
        b = in.nextInt();
        System.out.println(a + b);
   }
}
```

[Sample Code: Python]

```
import sys

a, b = (int(x) for x in sys.stdin.readline().strip().split())
sys.stdout.write("%d\n" % (a + b))
```

送分題 – solution

(30 分)

前言
比賽開始了！

趕快驗證一下，
網路是否設定正確?
上傳競賽程式是否順利?
檔案是否用 `STDIN` 輸入？
程式解答是否用 `STDOUT` 輸出？

都沒問題，30 分就到手了！ 繼續 … 衝！衝！衝！

問題敍述
試寫一程將輸入的兩個整數相加後輸出。

輸入格式
`A B`
兩個正整數，中間以空白區隔。

輸出格式
`C`
代表兩個正整數之和。

資料範圍
`A, B, C` 皆為小於 `9999999` 的正整數。

資料範例

輸入範例 1
```
2756 707
```

輸出範例 1
```
3463
```

輸入範例 2

```
23024 19109
```

輸出範例 2

```
42133
```

輸入範例 3

```
6253 14484
```

輸出範例 3

```
20737
```

範例說明：

兩個整數 2756 及 707 相加後輸出 3463。

補充說明：

以下為 STDIN /STDOUT 的方式輸入輸出的 Sample Code：

[Sample Code: C]

```
#include <stdio.h>

int main() {

     int a, b;
     scanf("%d %d", &a, &b);
     printf("%d\n", a+b);
     return 0;

}
```

[Sample Code: C++]

```
#include <iostream>
```

```
int main() {

     int a, b;
     std::cin >> a >> b;
     std::cout << a + b << std::endl;
     return 0;

}
```

[Sample Code: Java]

```
import java.util.Scanner;

public class solution {
    public static void main(String args[]) {
        Scanner in = new Scanner(System.in);
        int a, b;
        a = in.nextInt();
        b = in.nextInt();
        System.out.println(a + b);
   }
}
```

[Sample Code: Python]

```
import sys

a, b = (int(x) for x in sys.stdin.readline().strip().split())
sys.stdout.write("%d\n" % (a + b))
```

Q1 - String Rearranging

(10 points)

Description

Ming is a youngster who is passionate about string. Recently, he discovered a method to determine the similarity of two given strings on the Internet. The definition of the method is: Define s_1 as the first string, and $s_1[i]$ represents the i-th character of s_1. Define s_2 as the second string, and $s_2[i]$ represents the i-th character of s_2. The similarity of s_1, s_2 is the number of i such that $s_1[i] = s_2[i]$.

For example, the similarity of aabbcc and azbbzc is 4 since $s_1[1] = s_2[1]$, $s_1[3] = s_2[3]$, $s_1[4] = s_2[4]$ and $s_1[6] = s_2[6]$.

Ming wants to check that what's the maximum similarity if he can rearrange the second string. Since the number of permutations of a string may be enormous, Ming isn't capable to check all the permutations. Please write a program to help Ming.

Input Format

The first line contains a single integer N representing the length of the string.
The next line contains a string s_1 whose length is N and consists of only a to z.
The last line contains a string s_2 whose length is N and consists of only a to z.

Output Format

Please output the maximum similarity of s_1, s_2 after rearranging s_2.

Data Range

- $1 \leq N \leq 1000$

Input Example 1

```
5
abbcd
zbadp
```

Output Example 1

```
3
```

Input Example 2

```
4
yxxx
xxyx
```

Output Example 2

```
4
```

Input Example 3

```
3
ytp
gcc
```

Output Example 3

```
0
```

Example Explanation:

Example 1: We can rearrange the second string to `abzpd` so that the similarity of `abzpd` and `abbcd` is 3, and there's no permutation that can achieve higher similarity.

Example 2: We can rearrange the second string so that it's identical to the first string. Therefore, the similarity is the length of the string which is 4.

Example 3: It's impossible to rearrange second string to achieve the similarity that is greater than zero. So output 0.

問題 1 – 字串重排 (String Rearranging)

(10 分)

問題敘述

小明是一位很喜歡字串的少年，他最近在網路上發現一種計算兩個長度相同的字串的相似度的方法。該方法為：定義 s_1 為第一個字串，$s_1[i]$ 代表他的第 i 個字元，s_2 為第二個字串，$s_2[i]$ 代表他的第 i 個字元。兩個字串的相似度為有多少 i 使得 $s_1[i] = s_2[i]$。

舉例：aabbcc 與 azbbzc 的相似度為 4 ，因為 $s_1[1] = s_2[1]$ 、 $s_1[3] = s_2[3]$ 、 $s_1[4] = s_2[4]$ 、 $s_1[6] = s_2[6]$。

小明突發奇想，想測試看看若將第二個字串重新排列，那麼相似度最高能達到多少，但因為一個字串的排列數量非常多，小明無法測試全部的排列，因此希望你能寫一個程式幫助他。

輸入格式

第一行為一個整數 N 代表字串長度。
下一行輸入一個長度 N 且由 a 到 z 組成的字串 s_1。
最後一行輸入一個長度 N 且由 a 到 z 組成的字串 s_2。

輸出格式

請輸出在可以將 s_2 重新排列的情況下，兩個字串s_1, s_2 的所能達到的最高相似度為何。

資料範圍

- $1 \leq N \leq 1000$

輸入範例 1

```
5
abbcd
zbadp
```

輸出範例 1

```
3
```

輸入範例 2

```
4
yxxx
xxyx
```

輸出範例 2

```
4
```

輸入範例 3

```
3
ytp
gcc
```

輸出範例 3

```
0
```

範例說明

範例一，我們可以將第二個字串重新排列成為 abzpd，這樣他與 abbcd 的相似度為 3 。且不存在其他排列能讓相似度大於 3。

範例二，我們能將第二個字串重新排列後與第一個字串相等，因此相似度就是他們的長度也就是 4。

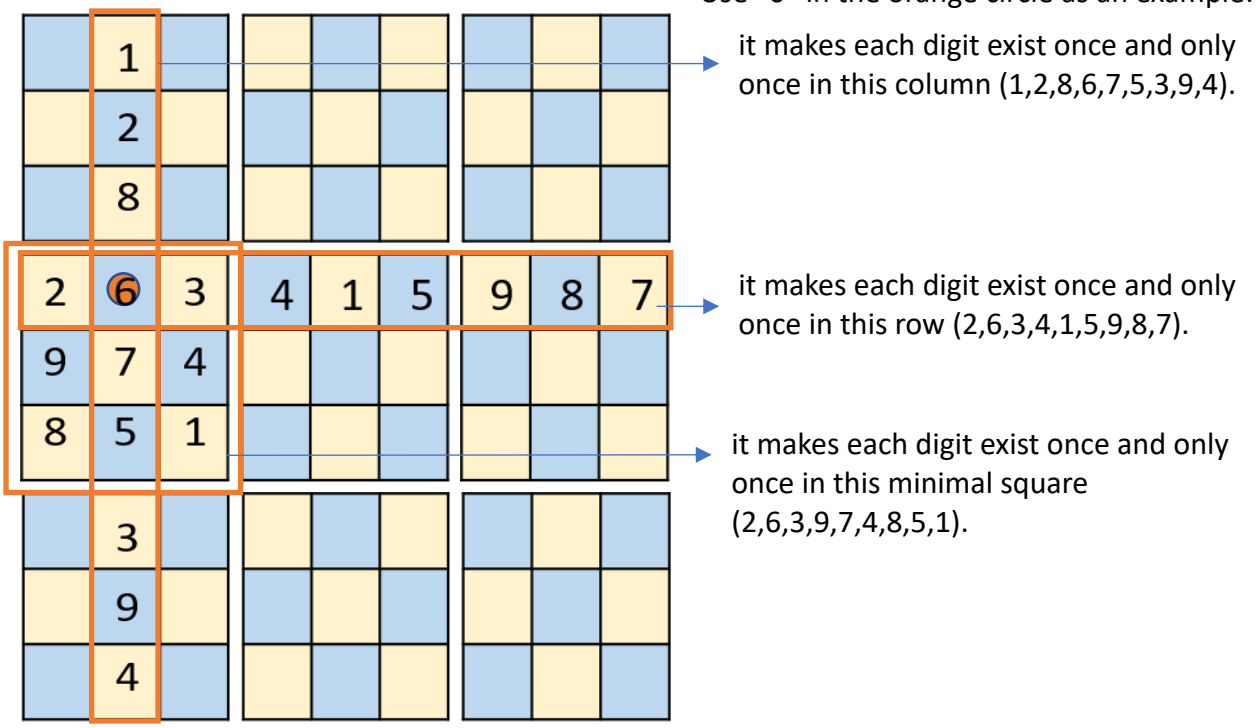
範例三，第二個字串的所有排列都不可能與第一個字串有大於 0 的相似度，因此輸出 0 。

Q2 - Sudoku

(15 points)

Description

Sodoku is a well-known game. First, it requires defines the minimal square's length (eg., 3 for 3x3 in below picture), unknown position's character, and all known characters. The rule of filling the unknown position is, the row (eg. 3x3=9 cells), the column (eg, 3x3=9 cells) and its minimal square, all characters must be unique in those 3 regions. Write a program to solve all unknown characters in the big square (eg., 9x9 in below).



Input Format

First line contains the length of the side for the minimal square (N), unknown character, and all known characters.
Second line to (N*N+1) lines, every line contains N*N characters (known characters or unknown characters)

Output Format

N*N lines, each line contains N*N characters (all known characters)

Data Range

1. All possible of first line is listed in below (N=2 or 3):
   - “2?ABCD”
   - “3#123456789”
2. Sudoku may have multiple solutions depends on puzzles. We will ensure the given puzzle only have one solution.

1st Example Input

```
3#123456789
3#65#84##
52#######
#87####31
##3#1##8#
9##863##5
#5##9#6##
13####25#
#######74
##52#63##
```

1st Example Output

```
316578492
529134768
487629531
263415987
974863125
851792643
138947256
692351874
745286319
```

1st Example Explanation

For example, the “1” is valid on row 0 and column 1 because it is unique in the following 3 areas.

- Row 0: 316578492
- Column 1: 128675394
- Minimal square 0: 316529487

2nd Example Input

```
2?ABCD
A?B?
???C
?B??
??D?
```

2nd Example Output

```
ACBD
BDAC
DBCA
CADB
```

3rd Example Input

```
3#123456789
25####9#7
##9####85
78##5#1##
###71#32#
9##563###
1#3#####4
495##6#18
6#2897###
#3######6
```

3rd Example Output

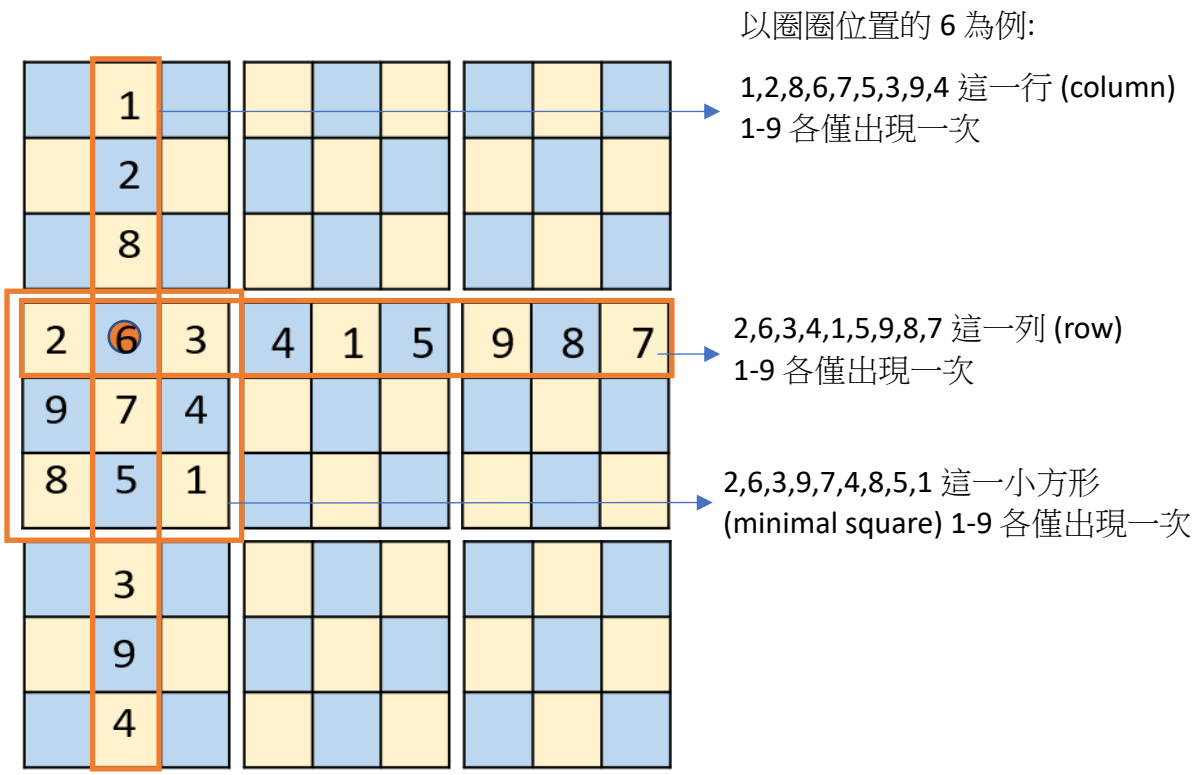
```
251638947
349271685
786459132
568714329
924563871
173982564
495326718
612897453
837145296
```

問題 2 – 數獨 (Sodoku)

(15 分)

問題敘述

數獨是一個常見的遊戲，首先先確立最小方形的邊 (如下圖: 3 for 3x3)、未知字元的符號以及所有已知的字元，填寫的規則是直行 (如下圖 3x3=9 格) 或縱行 (如下圖 3x3=9 格) 或是最小方形區域中 (如下圖 3x3=9 格) 不能有重複的項目，使得三個區域中每個已知字元都剛好出現一次，需要寫一個程式來解決一大方形 (如下圖 9x9) 中所有未知的項目。



輸入格式

第一行最小方形數 (N)、未知字元的符號和所有已知的字元。
第二行開始到第 (N*N+1) 行，每行都有 (N*N) 個字元。

輸出格式

第二行到第 (N*N) 行，每行都有 (N*N) 個字元，且沒有未知字元。

資料範圍

1. 第一行的所有可能值如下 (N 僅等於 2 或 3)
   - “2?ABCD”
   - “3#123456789”

2. 依不同的問題數獨可能會有多個解，但這裡我們會提供只有單一解的排列方式，以確保不依顯示順序而影響核對。

輸入範例 1

```
3#123456789
3#65#84##
52#######
#87####31
##3#1##8#
9##863##5
#5##9#6##
13####25#
#######74
##52#63##
```

輸出範例 1

```
316578492
529134768
487629531
263415987
974863125
851792643
138947256
692351874
745286319
```

範例 1 說明

第 1 行第 2 列的 “1” 是正確的, 因為下面三個區域它都是唯一的。

- 第 1 行: 316578492
- 第 2 列: 128675394
- 第 1 個小方形區域: 316529487

輸入範例 2

```
2?ABCD
A?B?
???C
?B??
??D?
```

輸出範例 2

```
ACBD
BDAC
DBCA
CADB
```

輸入範例 3

```
3#123456789
25####9#7
##9####85
78##5#1##
###71#32#
9##563###
1#3#####4
495##6#18
6#2897###
#3######6
```

輸出範例 3

```
251638947
349271685
786459132
568714329
924563871
173982564
495326718
612897453
837145296
```

Q3 - Warp Zone Repair

(15 points)

Description

In future days, there's a spatiotemporal teleportation machine, which can be teleported to the other time. One day, the existence of chaos – The Great Devil Electric Bear, shuffles the teleportation machines at various timelines together for fun. This is actually very dangerous. If user use the incorrect teleportation machine that should not exist at that time, it may cause the time paradox and make the user scatter and disappear. To avoid this dangerous situation happening, you, a spatiotemporal manager, need to keep all the spatiotemporal teleportation machines in their time.

However, because spatiotemporal teleportation machines of each timeline are always in the same place, it is impossible to directly transfer the teleportation machine to the correct time, otherwise the substances of the machine will overlap at the same coordinates. But on the other hand, you can spend a blaze rod to exchange two machines at any two time points.

So, the question is, what is the minimum number of blaze rods do you need to spend to make all the spatiotemporal teleportation of each timeline in the correct chronological order?

Input Format

The first line of the input contains two integers N, M, denoting the number of fire ships Little Y sent, and the number of Cao Cao's chained ships.

In the next M lines, the i-th line contains two integers x_i, t_i, denoting that the i-th fire ship will arrive on position x_i in time t_i.

The first line of the input contains one integer N, denoting that there's N spatiotemporal teleportation machine at various timeline requires you to transfer them to the correct time.

The second line of the input contains N positive integers, x_i, denoting that this machine should be exist in time x_i. Note that x_i satisfy the constraint that all the x_i are the integers from 1 to N and any two numbers are different from each other.

Output Format

Print how many blaze rods do you need to spend at least to make all the spatiotemporal teleportation of each timeline in the correct chronological order.

Data Range

- $1 \leq N \leq 1000000$

Input Example 1

```
4
1 2 3 4
```

Output Example 1

```
0
```

Input Example 2

```
3
2 3 1
```

Output Example 2

```
2
```

Input Example 3

```
4
2 1 4 3
```

Output Example 3

```
2
```

Example Explanation:

In example 1, all of machines are sorted in right chronological order, so you don't need an additional blaze rod for any exchange.

In example 2, you need to exchange 2 and 3 with blaze rod first, so that the sequence becomes 2 3 1. And then exchange 1 and 3 with the second blaze rod, so that sequence becomes 1 2 3.

In example 3, you need to exchange 2 and 1 with blaze rod first, so that the sequence becomes 1 2 4 3. And then exchange 4 and 3 with the second blaze rod, so that sequence becomes 1 2 3 4.

問題 3 – 時空傳送修復 (Warp Zone Repair)

(15 分)

問題敍述

在一個時間軸的同個固定座標上有很多時空傳送點可以傳送到其他時空去，但是某天可以穿梭在各個時空的渾沌存在——**電熊大魔王**為了有趣，將這些不同時空的時空傳送點打亂。這其實非常的危險：要是使用者使用了錯誤時空的時空傳送點，可能就會導致時間悖論，使使用者煙消雲散。為了避免這種危險的情況發生，身為時空修復員的你想要把這些時空傳送點送回正確的時空上。

不過，我們不能將時空傳送點直接傳送到其他時空點所在的位置，否則物質會在該座標重疊。但相對的，你可以選擇花一單位能量將任兩個時間點的傳送點等質交換。那麼請問，你最少要花多少單位能量才可以將所有時空傳送點送回正確的時空上呢？

輸入格式

輸入的第一行包含一個正整數 N ，代表接下來有 N 個時空傳送站需要送到正確的時空上。

接下來的一行有 N 個正整數分別代表 x_i ，表示第 i 個傳送點應該要被放在第 x_i 個時空上，並且 x_i 皆為 1 到 N 的數字，並且彼此不重複。

輸出格式

請輸出最少要花多少單位能量才可以將所有時空傳送點送回正確的時空上。

資料範圍

- $1 \leq N \leq 1000000$

輸入範例 1

```
4
1 2 3 4
```

輸出範例 1

```
0
```

輸入範例 2

```
3
2 3 1
```

輸出範例 2

```
2
```

輸入範例 3

```
4
2 1 4 3
```

輸出範例 3

```
2
```

範例說明

在範例一中，每個時空已經都在正確的位置上，所以不需要花任何能量來交換傳送點。

在範例二中，需要花一單位能量將 **2 3** 交換使各個傳送點變成 **3 2 1**，並再花一單位能量將 **1 3** 交換使其變成 **1 2 3**。

在範例三中，需要花一單位能量將 **2 1** 交換使各個傳送點變成 **1 2 4 3**，並再花一單位能量將 **4 3** 交換使其變成 **1 2 3 4**。

Q4 - Girl Group

(20 points)

Description

Practicing singing and dancing every day, Little B is a cadet in YTP Company, the most famous management company in Taipei. Today, she receives an announcement from her boss about the next girl group that YTP company wants to set up. Little B knows that YTP company has its own way to choose cadets. All cadets have their own ID, an integer from 1 to N, decided by when they entered the company. In order to unite the members of the girl group, YTP Company will only choose some cadets with continuous ID. Besides, each cadet has a grade based on their ability, which is represented by a lowercase Latin character. YTP Company thinks that the most important thing of a good girl group is balance, which means when a girl group includes cadets whose IDs are from l to r, then the grade of ID l must be the same as the grade of ID r, the grade of ID $l+1$ must be the same as the grade of ID $r-1$, and so on.

Now, Little B is wondering whether cadets with ID from l_i to r_i are possible to form a girl group. However, the grade of cadets may change, so the same cadets at different time may have different results, which makes the question more difficult. Can you write a program to help Little B to solve this question?

Input Format

The first line of the input contains two integers N, Q, denoting the number of cadets in YTP Company, and the number of operations which is either a grade change or a question asked by Little B.

The next line contains a string S with length N, denoting the initial grades of all cadets.

In the next Q lines, the i-th line is either $0\ k_i\ g_i$ or $1\ l_i\ r_i$. The former means that the grade of the cadet with ID k_i becomes g_i. The latter means that Little B wants to know whether cadets with IDs from l_i to r_i are possible to form a girl group.

Output Format

For each question, output a single line “YES” or “NO” (without “”) denoting whether those cadets are possible to form a girl group.

Data Range

- $1 \le N, Q \le 2 \times 10^5$
- S_i, g_i are all lowercase Latin characters.
- $1 \le\ k_i\ \le N$

- $1 \leq l_i \leq r_i \leq N$

Input Example 1

```
5 6
abcda
1 1 5
1 1 1
0 4 b
1 1 5
1 2 4
1 2 5
```

Output Example 1

```
NO
YES
YES
YES
NO
```

Input Example 2

```
5 6
abaaa
1 1 5
1 1 3
0 2 a
1 1 5
1 2 4
1 2 5
```

Output Example 2

```
NO
YES
YES
YES
YES
```

Input Example 3

```
9 11
stakataka
0 1 a
1 1 9
1 3 9
0 5 z
1 5 7
0 8 t
1 5 9
0 6 b
0 4 b
```

```
0 1 a
1 1 9
```

Output Example 3

```
NO
YES
NO
NO
YES
```

Example Explanation:

In example 1, the events happen with the following order:

- “abcda” is not balance.
- “a” is balance.
- The whole grade sequence changes to “abcba”.
- “abcba” is balance.
- “bcb” is balance.
- “bcba” is not balance.

In example 2, the events happen with the following order:

- “abaaa” is not balance.
- “aba” is balance.
- The whole grade sequence changes to “aaaaa”.
- “aaaaa” is balance.
- “aaa” is balance.
- “aaaa” is balance.

In example 3, the events happen with the following order:

- The whole grade sequence changes to “atakataka”.
- “atakataka” is not balance.
- “akataka” is balance.
- The whole grade sequence changes to “atakztaka”.
- “zta” is not balance.
- The whole grade sequence changes to “atakztata”.
- “ztata” is not balance.
- The whole grade sequence changes to “atakzbata”.
- The whole grade sequence changes to “atabzbata”.
- The whole grade sequence changes to “atabzbata”.(In fact, the original grade of cadet 1 is “a”, so there is nothing change.)
- “atabzbata” is balance.

問題 4 – 女團夢想 (Girl Group)

(20 分)

問題敍述

作為一個在臺北最有名的 YTP 經紀公司工作的練習生，小 B 每天都在練習唱歌和跳舞。今天，他接到老闆發的公告，提到了公司想要成立一個新的女子團體。小 B 知道 YTP 經紀公司有自己選擇練習生的方式。所有練習生都有自己的編號，是一個從 1 到 N 的正整數，由他們進入公司的時間決定。為了讓團體的成員能夠團結，YTP 經紀公司只會選擇連續編號的練習生來組成女團。除此之外，每個練習生都會根據他們的能力被標上小寫英文字母表示的等第。YTP 經紀公司認為一個好的女子團體最重要的事情是平衡，意思是當我們令這個女子團體是由編號 l 到編號 r 的練習生組成，那麼編號 l 的練習生和編號 r 的練習生等第就要相同，編號 $l+1$ 的練習生和編號 $r-1$ 的練習生等第就要相同，依此類推。

現在，小 B 很好奇編號 l_i 到編號 r_i 的練習生是否有可能組成一個女團。然而，每個練習生的等第可能會改變，因此同樣的人選可能會在不同時間有不同的結果，這讓問題變得更難了。你能寫一支程式幫助小 B 解決這個問題嗎？

輸入格式

輸入的第一行包含兩個正整數 N, Q ，代表 YTP 經紀公司的練習生數量以及事件數量，每個事件可能是有人的等第改變或者是小 B 想知道某些練習生是否有可能組成女子團體。

下一行包含一個長度為 N 的字串 S 代表所有練習生的等第。

接下來的 Q 行，第 i 行會形如 0 k_i g_i 或 1 l_i r_i。前者代表編號為 k_i的練習生的等第變成 g_i ；後者代表小 B 想知道編號從 l_i 到 r_i 的練習生是否有可能組成女子團體。

輸出格式

對於每筆詢問輸出一行 “YES” 或 “NO” （不包含 “”）表示那些練習生是否有可能組成女子團體。

資料範圍

- $1 \le N, Q \le 2 \times 10^5$
- S_i, g_i 都是小寫英文字母。
- $1 \le k_i \le N$
- $1 \le l_i \le r_i \le N$

輸入範例 1

```
5 6
abcda
1 1 5
1 1 1
0 4 b
1 1 5
1 2 4
1 2 5
```

輸出範例 1

```
NO
YES
YES
YES
NO
```

輸入範例 2

```
5 6
abaaa
1 1 5
1 1 3
0 2 a
1 1 5
1 2 4
1 2 5
```

輸出範例 2

```
NO
YES
YES
YES
YES
```

輸入範例 3

```
9 11
stakataka
0 1 a
1 1 9
1 3 9
0 5 z
1 5 7
0 8 t
1 5 9
0 6 b
0 4 b
0 1 a
1 1 9
```

輸出範例 3

```
NO
YES
NO
NO
YES
```

範例說明

在範例一中，以下的事件依序發生：

- “abcda” 是不平衡的。
- “a” 是平衡的。
- 成績序列變成 “abcba”。
- “abcba” 是平衡的。
- “bcb” 是平衡的。
- “bcba” 是不平衡的。

在範例二中，以下的事件依序發生：

- “abaaa” 是不平衡的。
- “aba” 是平衡的。
- 成績序列變成 “aaaaa”。
- “aaaaa” 是平衡的。
- “aaa” 是平衡的。
- “aaaa” 是平衡的。

在範例三中，以下的事件依序發生：

- 成績序列變成 “atakataka”。
- “atakataka” 是不平衡的。
- “akataka” 是平衡的。
- 成績序列變成 “atakztaka”。
- “zta” 是不平衡的。
- 成績序列變成 “atakztata”。
- “ztata” 是不平衡的。
- 成績序列變成 “atakzbata”。
- 成績序列變成 “atabzbata”。
- 成績序列變成 “atabzbata”（因為第 1 個人的等第本來就是 a，因此其實沒有變化）。
- “atabzbata” 是平衡的。

Q5 - Budget Cut

(20 points)

Description

The Tainlong Kingdom is a country with a complex transportation system. In order to develop tourism in the capital, Rep. City councilor, Hachi put forward M proposals for direct subways from the capital to other cities. After some precise calculations, it was found out that the shortest distance between the capital and some other city was indeed pulled. But clever Rep. Man saw something was wrong at a glance. He found out that even if some proposed subways were not established, the distance of the new plan will not be affected. In able to point out the problems at once, Rep. Man decided to find out the most subways path that can be removed synchronously without altering the benefits of the new proposal. As Rep. Man's best staff, can you tell him how many such subway paths can be removed together?

Input Format

The first line contains N, M, S- the number of cities, roads, and subways proposals.
The Following M lines represent the original path of the country.
Each line contains u, v, w - a bidirectional path of length w connecting u and v cities.
The Following S lines are the proposals.
Each line contains v, w - a bidirectional subways connecting city 1 (the capital) and city v with length w.

Output Format

Print the max number of proposals that can be removed together without changing the benefits of the Rep. Hachi's plans.

Data Range

- $1 \leq N, M, S \leq 100000$
- $1 \leq u, v \leq N$
- $1 \leq w \leq 1000000$

範例說明

Input Example 1

```
3 1 2
2 3 3
2 4
3 1
```

Output Example 1

```
1
```

Input Example 2

```
3 2 2
1 3 2
3 2 1
3 2
2 4
```

Output Example 2

```
2
```

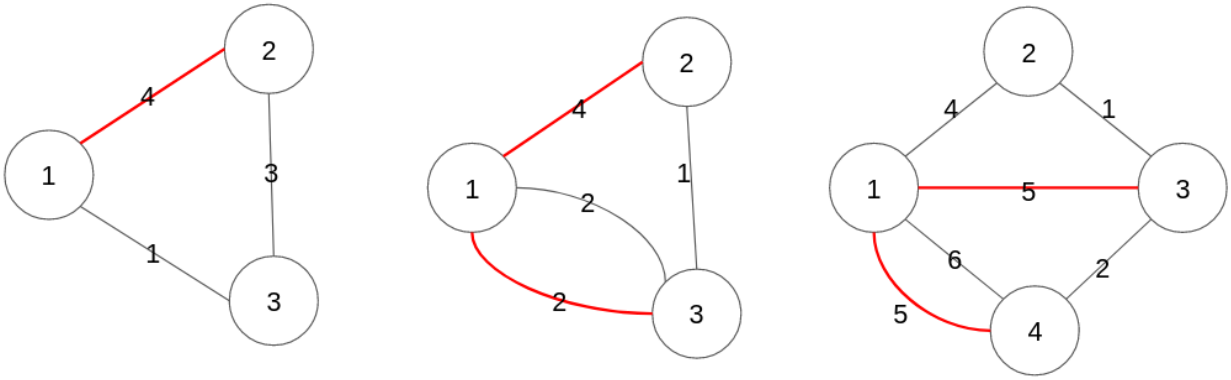
Input Example 3

```
4 4 2
1 2 4
2 3 1
3 4 2
1 4 6
3 5
4 5
```

Output Example 3

```
1
```

Example Explanation:



For example 1, the only proposal can be removed.
For example 2, both proposals do not shorten the distance between the cities and the capital.
For example 3, removing the first proposal alone does not cause any changes of the shortest distance from the capital, but removing the second proposal will increase the distance between the 4th city and the capital.

問題 5 – 砍預算

(20 分)

問題敘述

天龍國是一個交通複雜的國家，其中首都城中市為了發展觀光，提出了一些直達其他縣市的地下鐵。今天哈其議員在議會提出了 M 個直達地下鐵的提案，在經過大家的一番計算，發現確實拉進了城中市與一些縣市的距離。但是聰明的潘議員，一眼就看出有些不對，他發現有些直達路線，就算不建立，也不會改變任城中市與任何一個縣市的距離。為了能夠一舉指出哈其議員的弊處，潘議員決定蒐集最多同時移除也不會比原本提案的距離更差的直達路線。身為潘議員幕僚的你，有辦法告訴潘議員他最多可以從提案中找到幾個這樣的直達路線嗎。

輸入格式

第一行有三個數字 N, M, S分別代表城市、道路、和直達地下鐵方案的數量
接下來 M行，每行代表一個雙向的道路，有三個正整數 u, v, w 代表這個距離 w的道路連接 u, v兩個城市
最後 S 行，每行代表一個雙向直達地下鐵，有兩個整數 u, w代表連接首都（1 號城市）和 u 城市，距離是 w。

輸出格式

輸出一個整數代表最多可以同時移除幾個直達地下鐵方案，而不影響首都到任何一個城市的最短距離。

資料範圍

- $1 \leq N, M, S, \leq 100000$
- $1 \leq u, v \leq N$
- $1 \leq w \leq 1000000$

輸入範例 1

```
3 1 2
2 3 3
2 4
3 1
```

輸出範例 1

```
1
```

輸入範例 2

```
3  2  2
1  3  2
3  2  1
3  2
2  4
```

輸出範例 2

```
2
```

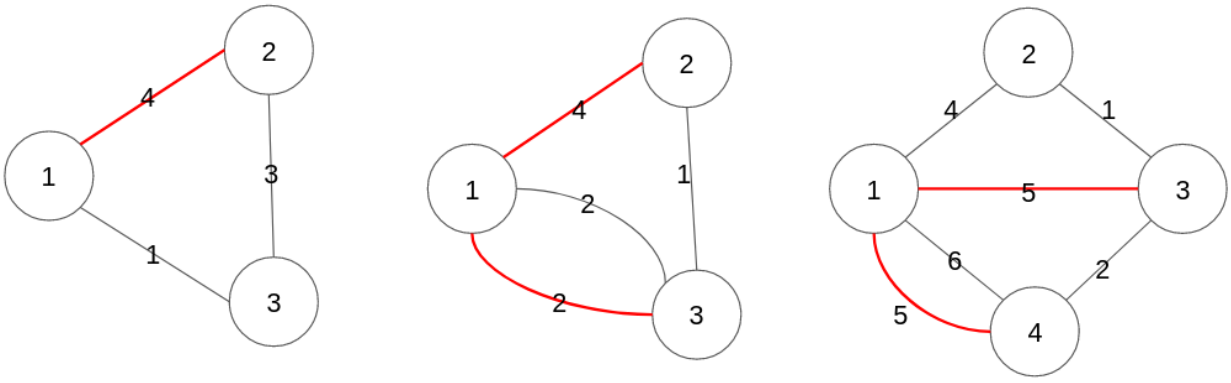
輸入範例 3

```
4  4  2
1  2  4
2  3  1
3  4  2
1  4  6
3  5
4  5
```

輸出範例 3

```
1
```

範例說明



第一個範例 (上圖左)，可移除第一個方案，連結首都 1 與城市 2 的地鐵提案。
第二個範例 (上圖中)，兩個提案都不會讓其它城市(2 或 3)跟首都 1 的距離縮短
第三個範例 (上圖中)，可移除第一個方案，不會對任何最短路造成影響，但移除第二個方案會讓 1 和 4 的距離變遠

Q6: Abundance Sum

(25 points)

Description

A number N is **perfect** if it is equal to the sum of its proper divisors (positive divisors which are strictly less than it). For example, $6$is a perfect number, because it has three positive divisors: $1, 2,$ and 3 (note that 6 itself is not counted). And since 6 itself is equal to $1 + 2 + 3$, it is a perfect number. We can denote the **abundance** of a number N as$\Delta(N)$, and define it as the difference between N and its proper divisors:

$$\Delta(X) = X - \sum_{Y|X, Y<X} Y$$

If $\Delta(X) = 0$, then X is a perfect number; if it is positive, then we call such a number **abundant**; otherwise, we call it **deficient**. There are many mysteries surrounding these numbers, many of which have been around since Pythagorean times, for example:

* Do there exist **odd** perfect numbers?
* Does there exist a number N such that $\Delta(N) = -1$?

But the problem we have at hand is a lot easier: given a closed interval $[L, R]$, your task is to find the sum S of all $\Delta(N)$ for $N \in [L, R]$. That is, find

$$S = \sum_{N=L}^{R} \Delta(N)$$

To prevent integer overflows, please output S modulo the prime $10^9 + 7$. Good luck!

Note: this problem **gives out points for partial solutions**! Try your hand at the smaller testcases before moving on the larger ones!

Input Format

The input will have only one line, with two integers L, R.

Output Format

Please output the answer on one line.

Data Range

For all given inputs, $1 \leq L \leq R \leq 10^{12}$.

Additionally, this problem gives out scores for partial solutions:

- $R \leq 10^4$, worth6 points.
- $R \leq 10^6$, worth 9 points.
- $R \leq 10^{12}$, worth 10 points.

Input Example 1

```
2 5
```

Output Example 1

```
8
```

Input Example 2

```
6 6
```

Output Example 2

```
0
```

Input Example 3

```
1 10000
```

Output Example 3

```
17753986
```

Example Explanation:

For Example 1: $2 - (1) + 3 - (1) + 4 - (1 + 2) + 5 - (1) = 8$.
For Example 2: $6 - (1 + 2 + 3) = 0$.

問題 6 – 陰晴圓缺 (Abundance Sum)

(25 分)

問題敍述

一個數字 N 為「完美數」若且唯若它的所有小於它的正因數的和恰等於那個數字。舉例來說，6 的因數有 1、2、3，且 $1+2+3=6$，所以 6即為一個完美數。我們可以定義一個「盈度」函數 $\Delta(X)$，定義為 X 與 X 的所有因數的差：

$$\Delta(X) = X - \sum_{Y|X, Y<X} Y$$

如果 $\Delta(X) = 0$，X 就是完美數；而如果 $\Delta(X) > 0$，則這個數字稱為**盈數**，$\Delta(X) < 0$ 的則稱為**虧數**。完美數有許多圍繞著它的謎團，不少從古希臘的畢達哥拉斯時期就開始有人提出了，包括：

- 是否存在一個奇的完美數？
- 是否有數字 N 使得 $\Delta(N) = -1$？

不過，現在想要解決的問題比較簡單：給你一個區間 $[L, R]$，你能夠找到

$$S = \sum_{N=L}^{R} \Delta(N)$$

嗎？因為數字可能會非常大，請輸出 S 除以質數$10^9 + 7$ 的餘數即可。

溫馨提醒：**這一題有部分分數**，不妨在嘗試滿分解前先解出範圍比較小的測試資料吧！

輸入格式

輸入將有一行，為兩個數字 L 與 R。

輸出格式

請輸出一個數字，代表答案。

資料範圍

對於所有的輸入資料，皆有 $1 \le L \le R \le 10^{12}$。

此外，此題有部分給分：

- 第一部份測資通過給 6 分，測試資料滿足 $R \le 10^4$。
- 第二部份測資通過給 9 分，測試資料滿足 $R \le 10^6$。
- 第三部份測資通過給 10 分，測試資料滿足 $R \le 10^{12}$。

輸入範例 1

```
2 5
```

輸出範例 1

```
8
```

輸入範例 2

```
6 6
```

輸出範例 2

```
0
```

輸入範例 3

```
1 10000
```

輸出範例 3

```
17753986
```

範例說明

範例輸入 1: $2 - (1) + 3 - (1) + 4 - (1 + 2) + 5 - (1) = 8$。
範例輸入 2: $6 - (1 + 2 + 3) = 0$。